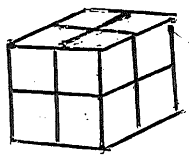
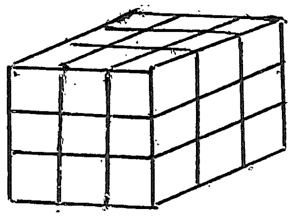
WARM UP

1. A cube of order n is made up of $n \times n \times n$ smaller cubes, all the same size.

   We are interested in the number of smaller cube faces that are concealed <u>within</u> the larger cube.

eg	n	2	3
			
number of smaller cubes		8	27
small cube faces on outside		24	54
small cube faces concealed within		24	108

   Obtain a general formula for the number of small cube faces concealed within a cube of order n.

2. Early one morning Jack planted the bean that he had taken home after the pantomime. He watched the resulting beanstalk grow each day.

   On the second day the amount the beanstalk grew was equal to the height it was at the start of that second morning.
   Next, on the third day the amount the beanstalk grew was equal to one half of the height it was at the start of the third morning.
   Over the fourth day the amount the beanstalk grew was equal to one third of the height it was at the start of the fourth morning. And so on….
   Assume that this pattern of growth carries on.

   At the end of the first week's growth the beanstalk's height was fourteen inches.

   How tall was the beanstalk at the end of its' first fortnight?
   How tall was it at the start of the morning on its' first (non-leap) year birthday?

3. Identify plausible next members in each of the following sequences.

(i)	1	2	3	6	7	8
(ii)	1	8	11	18	80	81

   It might help you if you spell out your reasoning clearly.

Stages in Problem Solving – What to do when stuck!

- It's not what you do, it's the way that you do it …

Approach

When faced with a **problem**, either (i) as a statement with a proposition to be derived, developed or proved or (ii) as a difficulty that is preventing you from getting from where you are to where you want to get to, you may find it helpful to adopt a systematic approach.
You need the approach that is best for yourself and best for the particular problem. Your approach does not have to be identical to the one described here.

Abilities required

- Recall :to recollect theorems, techniques and facts.
- Comprehension :to interpret and express ideas in different ways.
- Application :to use techniques both in familiar and new situations.
- Analysis :to break given information into comprehensible parts.
- Synthesis :to join ideas together.
- Evaluation :to compare one set of ideas with another set and make a judgement about a solution according to some criteria.

Stages

Stages	Key Steps (Check List)
• *Get started*	:**Understand** the problem Know what you are given and where you want to get to.
• *Plan*	:**Introduce** diagram, notation, model. < **Recall** possible useful theorems and techniques.
• *Mull*	:Think.
• *Keep going*	
• *Gain insight*	:**Conjecture** **Test** conjecture **Justify/prove** conjecture **Generalise** (if appropriate) Go back to Plan if necessary >
• *Be sceptical*	:**Check** solution
• *Contemplate*	:**Review** (Implications? Better methods? Extensions?)

What to do when stuck!

- Consider a particular case/ simpler case/ special case for clues as how to proceed.
- Consider a simpler/ more accessible problem.
- Consider a similar/ analogous problem.

Example

It is believed that if the difference between two given positive integers is even then their product is the difference of the squares of two integers.

Particularise

$3\times1 = 3 = 2^2-1^2 = (2+1)(2-1)$
$5\times3 = 15 = 4^2-1^2 = (4+1)(4-1)$
$6\times4 = 24 = 5^2-1^2 = (5+1)(5-1)$
$5\times1 = 5 = 3^2-2^2 = (3+2)(3-2)$

Recognise pattern

Conjecture

When the difference between the given integers is 2, the difference in squares factorises (x+1)(x-1).
When the difference between the given integers is 4, the difference in squares factorises (x+2)(x-2).
This suggests that when the difference between the given integers is 2k, the difference of squares factorises (x+k)(x-k). What is x interms of the given integers?

Test

Try example in which the difference between the integers is 6

Generalise

Effectively, working with a=average of the two given integers and k=half the difference, we know that $(a+k)(a-k)=a^2-k^2$

Check solution

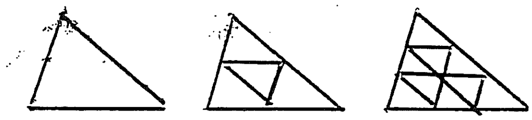
Particular examples above fit this pattern.

The problem is solved and the "belief" is confirmed

Exercises & Possible Solutions *(Student Copy does not include solutions)*

1. Each side of a triangle is marked with points that divide the side into (integer) n equal segments. A set of smaller triangles is formed by lines constructed through these points and parallel to the other sides of the larger triangle. The counts C_n of the smaller non-overlapping triangles for various values of n are shown.

n = 1 2 3

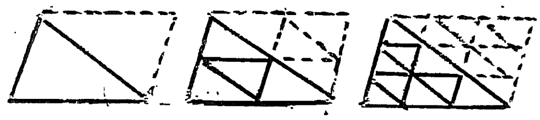


C_n = 1 4 9

Conjecture the count for the general value n. Prove your conjecture.

Solution

Particularise
As given
Conjecture
Count $C_n = n^2$
Test
OK for $n=4$
May be stuck
Think of similar/analogous problen involving $C_n = n^2$
Eg Squares, Rhombuses. How to invoive triangies?

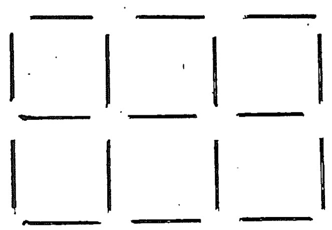


Solution follows.

(*Alternatively for triangles, note that "adding a line" yields*
$C_{n+1}=C_n+(2n+1)$
Solution follows)

Check
Particular examples above fit this pattern.
Problem is solved.

2. The number of matchsticks required to form a rectangle consisting of 2×3 squares, as shown, is 17.



How many matchsticks are required to form a rectangle consisting of
(i) 4×6 squares?
(ii) $m\times n$ squares?

Solution

(i) By counting 58
(ii)May be stuck

Simplify. Look for pattern.

Pattern made up of left side edge (m matches) and top edge (n matches) together with the bottom edge and right side (2 matches) for each of the specified number of squares.

The total required here is thus $m + n + 2mn$

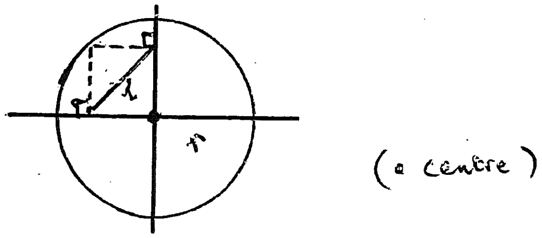
Check solution.
Particular examples above fit this equation.

3. What two whole numbers, neither containing zeros, when multiplied together equal 1000000000?

Solution

$10 = 2 \times 5$
$1000000000 = 10^9 = (2 \times 5)^9 = 2^9 \times 5^9 = 512 \times 1953125$

4. For the given construction, express ℓ in terms of r.

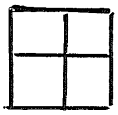


5. Under what circumstances is the following true?
"The day before yesterday she was 14. Next year she will be 17."

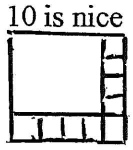
Solution
Today is the 1st January of the current year.
The day before yesterday was 30th December .She was still 14.
Yesterday, 31st December, was her 15th birthday.
On 31st December of the current year she will become 16.
On the same date nexy year she will be 17.

6. An integer number n is a 'nice' number if a square can be partitioned (split up) into n non- overlapping squares.
For example,

4 is nice



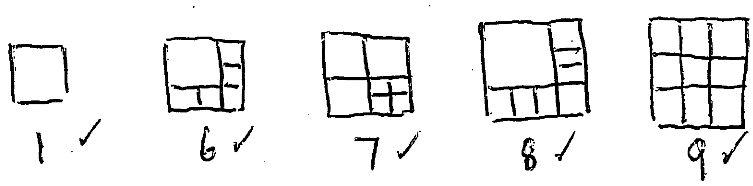
10 is nice



What numbers between 1 and 10 inclusive appear to be not nice?

Solution

Answer; 2, 3, 5



In conclusion:

Some thoughts

- If appropriate, attempt to avoid or to work around your problem.
- When approaching problem solving be methodical. Management texts suggest you ask the questions "Where are we now?
  Where do we want to get to?
  How can we get there?
  How do we know that we have arrived?"
- Imagine that you have engaged a very expensive private tutor paid by the minute. As if before a meeting with your tutor, work out exactly what you would want to say and what you would want to ask.
  "The best way to understanding is through trying to explain".
  Often this proves valuable in forcing your thoughts into some coherent order which will lead you to a solution without recourse to external assistance.
  Think of this as "self-tutoring".
- If "self-tutoring" does not work then seek assistance.
  "A problem shared is a problem halved"/ "Two heads are better than one".
  Another person might bring different experience, suggest a different perspective/ point of view, or offer a different skill set. Indeed to them your problem might be no problem at all.
- Above all, remember the "eating the elephant problem" advice.
  "How do you eat an elephant?
  You take bite size pieces and chew on each of them in turn!"

Two more problems

1. In the league table below, each team played each other team once: two points were awarded for a win, one for a draw. What was the result of the match between City and United?

	P	W	D	L	GF	GA	Points
City	3	2	1	0	4	1	5
Rovers	3	2	0	1	3	1	4
Albion	3	0	2	1	0	2	2
United	3	0	1	2	1	4	1

2. Six circular discs, three green, three yellow and three red, are to be placed on a 3 by 3 square arrangement so that the ventral one touches exactly four others. Those on the edges touch those discs adjacent to them. In addition, each green disc must touch at least one yellow one, each yellow one must touch at least one red one, and each red one must touch at least one green one.
   In how many ways can this be done?

Answers

1. City won 3-1.

2. If you think the answer is 12 then you can be only partly satisfied with your efforts as the answer is in fact 36.

"Maths Club" Session: Saturday, 31st January 2004.

Postscript Commentary

Handout-material

The edition of the material, as shown above, is the "staff" version as made available to "helpers" at the Maths Club presentation. The edition available for participants' use during the presentation was as above but redacted with the italicised sections removed.

Influences

Two books were major influences on the content of the presentation:

(i) "How to Solve It" , George Polya, Penguin Science, 1945.
A facsimile copy is available at
https://notendur.hi.is/hei2/teaching/Polya_HowToSolveIt.pdf

(ii) "Thinking Mathematically" 2nd edition, John Mason et al, Prentice Hall, 2010.

Effectiveness of Use of Checklists and Acceptance as the Norm in various fields including, in particular, Surgical Operations.

(i) Car Driver Checklist: "Mirror, signal, manoeuvre" etc
(ii) Air Pilot Pre-take-off Checklist
(iii) Pre-Surgical Operation Checklist

In the past ten years or so Atul Gawande of Harvard Medical School has championed the use of pre-surgery checklists and has collected dramatic evidence of the benefits of their implementation. There are many examples of reports of the outcomes of his investigations. One of the shorter reports is from Center News Magazine: "Atul Gawande Draws Large Crowd for President's Special Lecture"Monday, February 1, 2010."
http://www.mskcc.org/magazine/february-2010/atul-gawande-draws-large-crowd-president-special-lecture.

Adoption of checklists is not a sign of personal "weakness", only a sign of personal "realism"!

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